**FAST-NUCES Assignment # 1 Computer Modeling and Simulation**

**Q.1 Newton’s Law of Heating and Cooling** states that the rate of change of the temperature (T) with respect to time (t) of an object is proportional to the difference between the temperatures of the object and of its surroundings. Suppose the temperature of the surroundings is 25 ˚C. **[2+2+3+5+2+4+2=20 marks]**

a. Write the differential equation that models Newton’s Law.

b. Solve this equation for T as a function of time t using the analytical method.

c. Create system dynamic diagrams in NetLogo.

d. Write a simulation program using the Euler Method and plot the temperature over time using any programming language.

e. Plot the error, using a program, between the actual solution (determined in step a) and the solution determined via Euler method by taking different values for the time step.

f. It was a moderate day in April with room temperature at 25 ˚C. Detective Ali Imran arrived at the crime scene in a hotel room to find inspector Jamshed leaning over the body. The inspector said there were several suspects. If they knew the exact time of death, then they could narrow the list. Ali Imrran took out a thermometer and measured the temperature of the corpse: 28 ˚C. The normal body temperature of a living person is 37 ˚C. He then left for lunch. Upon returning at 1:00pm, he found the body temperature to be 27˚ C. How long ago was the murder committed from discovery of the body?

g. Suppose we are performing a simulation using a step size of 0.004 h. Using the decay rate from part a, determine the temperature at the end of the first three time steps after discovery of the body.

**Q. 2.** **Gompertz Model for Tumor Growth** has the following ordinary differential equation.

dN/dt = kN ln( M / N) **[2+5+3+2=12 marks]**

where,

N(0) = 1000 cells is the initial condition

N is the number of cancer cells

k is the proliferation ability of cancer cells (10% per month)

M = 1 million is the carrying capacity

For this ODE, do the following:

a. Create system dynamic diagrams in NetLogo.

b. Write a simulation program using the Euler Method and plot the temperature over time using any programming language. Assume a suitable value for the time step.

c. Determine the solution for the above equation using the analytical method. Follow the following steps to find the analytical solution.

i. Make the substitution u = ln(M/N) in the Gompertz equation to eliminate N and convert the equation to be in terms of u.

ii. Using calculus, solve the transformed differential equation for u.

iii. Using the relationship between u and N from Part b, convert your answer from Part c to be in terms of N. The result is the solution to the Gompertz differential equation.

d. Plot the error, using a program, between the actual solution (determined in step c) and the solution determined via Euler method by taking different values for the time step.

**Q. 3.** A feature of radioactive decay is that the amount of a radioactive substance decreases at a rate proportional to the current amount of the substance. The *half life* of a substance is the amount of time it takes for half of a given amount of substance to decay. The half life of carbon-14 is approximately 5730 years. If an ancient object has a carbon-14 amount that is 20% of the original amount, how old is the object?  **[3 marks]**

Q. 4. Army ants on a 17-km2 island forage at a rate of 1500 m2 /day, clearing the area almost completely of other insects. Once the ants have departed, it takes about 150 days for the number of other insects to recover in the area. Assume an initial number of 1 million army ants and a growth rate of 3.6%, where the unit of time is a week. Model the population.  **[5 marks]**